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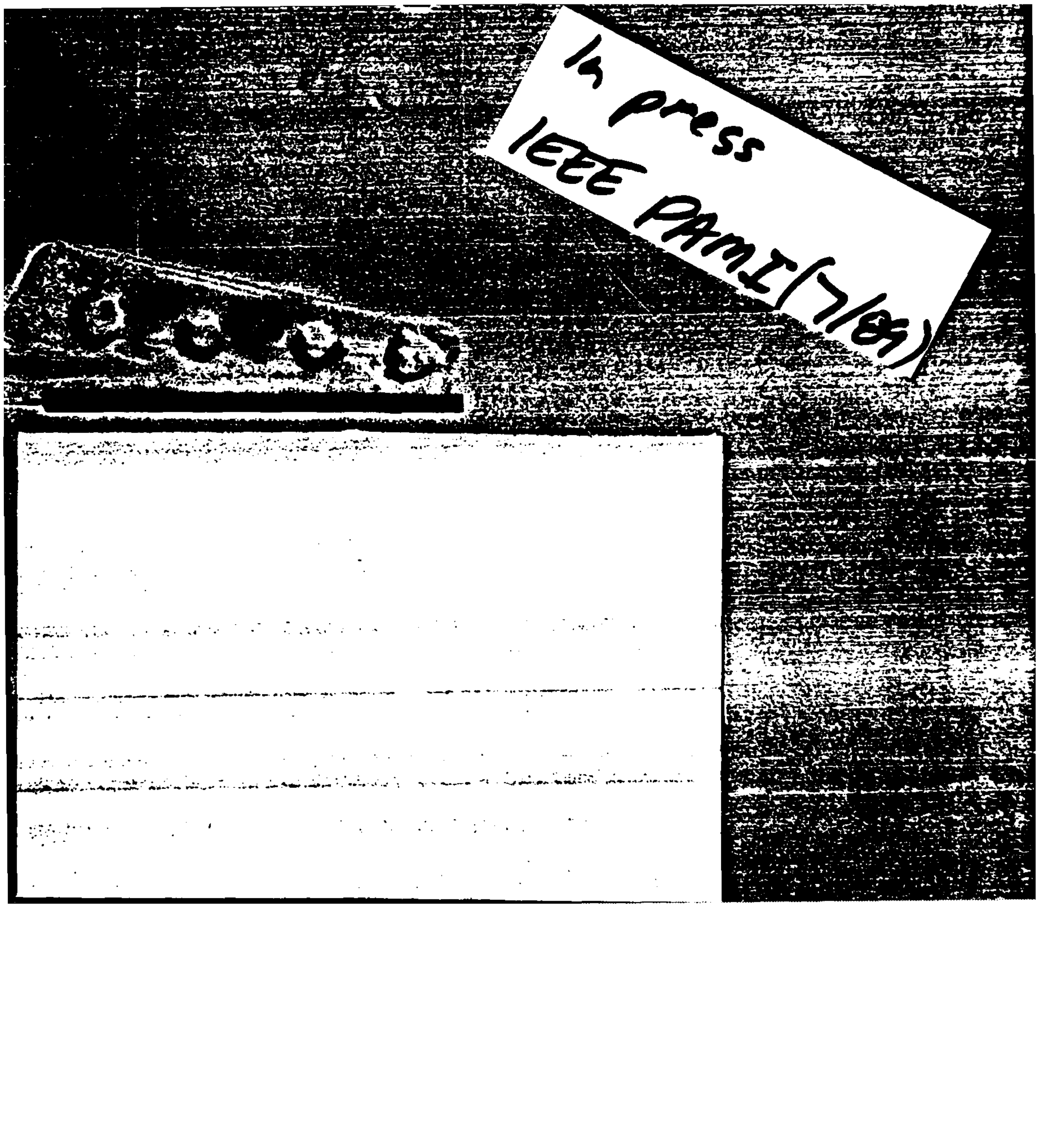
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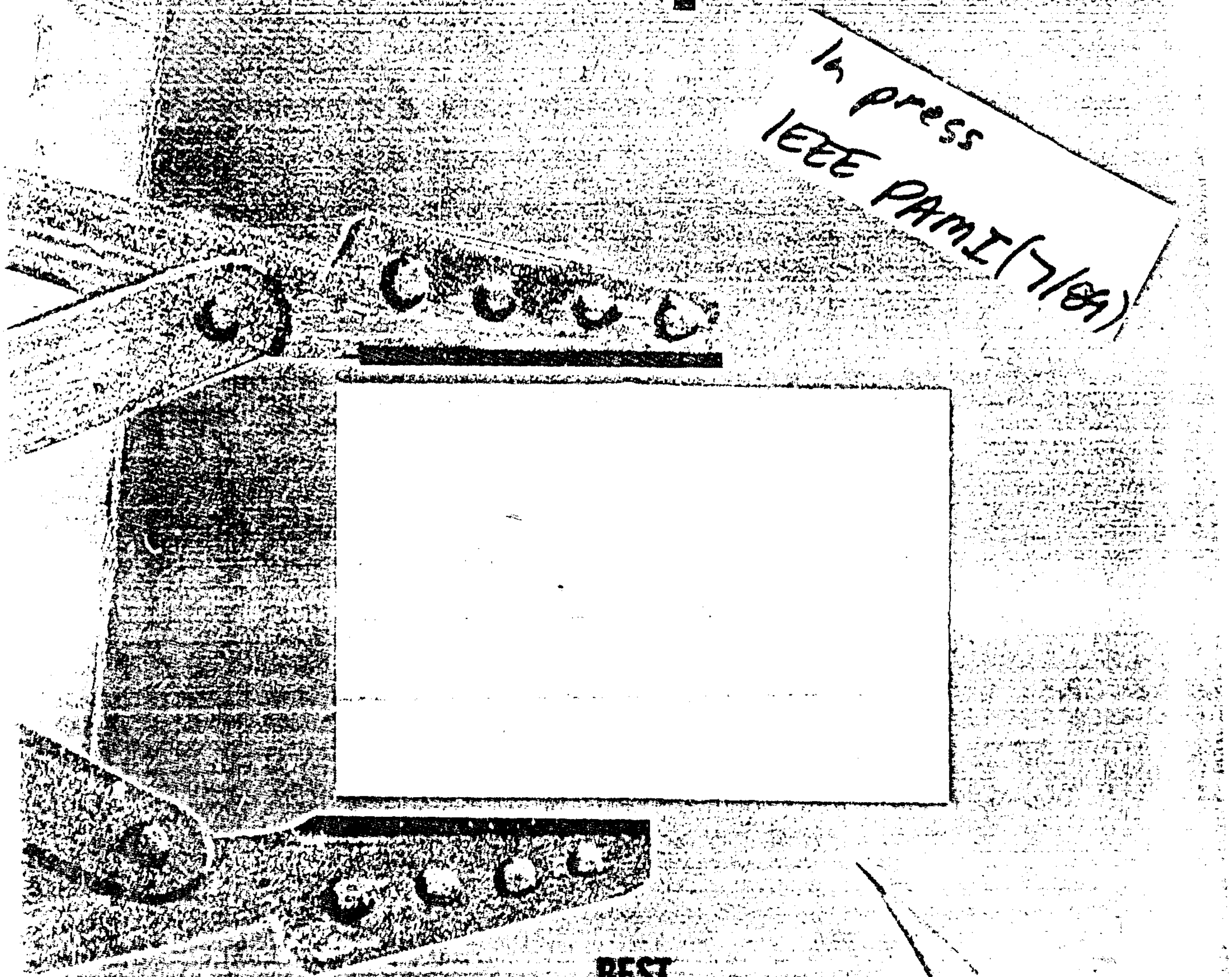

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New York University
Courant Institute of Mathematical Sciences

Computer Science Division
251 Mercer Street New York, N.Y. 10012

The Generalized Map Makers Problem: Optimal Flattening of Polyhedral Surfaces

by

Eric L. Schwartz †‡
Alan Shaw ‡
Estarose Wolfson ‡

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† New York University
Dept. of Computer Science
Courant Institute of Mathematical Sciences
251 Mercer Street
New York, New York 10012

‡ Computational Neuroscience Laboratories
Dept. of Psychiatry
NYU Medical Center
550 First Ave
New York, N.Y. 10016

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A NUMERICAL SOLUTION TO THE GENERALIZED MAP-MAKER'S PROBLEM: FLATTENING NON-CONVEX POLYHEDRAL SURFACES

Eric L. Schwartz

Alan Shaw

Estarose Wolfson

Robotics Research
Courant Institute
Department of Computer Science
715 Broadway
New York, N.Y. 10003

Computational Neuroscience Laboratories
New York University School of Medicine
Department of Psychiatry
550 First Avenue
New York, N.Y. 10016

ABSTRACT

~~Our~~ concern is to "unfold" and flatten the curved, convoluted surfaces of the brain in order to study the functional architectures and neural maps embedded in them. In order to do this, it is necessary to solve the general map makers problem for representing curved surfaces by quasi-isometric planar models. This algorithm has applications in areas other than computer aided neuroanatomy, such as robotics motion planning and geophysics.
The algorithm ~~we have~~ written maximizes the goodness of fit of distances in these surfaces, to those in a planar configuration of points. ~~We~~ illustrate this algorithm with a flattening of monkey visual cortex, which is an extremely complex, folded surface. ~~We find~~ distance errors in the range of several percent, with isolated regions of larger error, for the class of cortical surfaces which we have so far studied.

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Eric L. Schwartz

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Robotics Research
Courant Institute
Department of Computer Science
715 Broadway
New York, N.Y. 10003

Computational Neuroscience Laboratories
New York University School of Medicine
Department of Psychiatry
550 First Avenue
New York, N.Y. 10016

INTRODUCTION

The map-maker's problem is to find a flat representation of a curved surface: for example, the surface of the earth. Classical map making has been restricted to the relatively simple spherical surface of the earth. In the case that the surface of interest is complex, and possibly non-convex, there are no known methods of finding quasi-isometric planar representations.

The solution to this problem is of importance to computer aided neuro-anatomy, since it is often desired to view the surface of various cortical areas in a planar model. Primate cortex is highly convoluted, and provides one of the more complex surfaces encountered in practical applications.

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Motion planning in robotics is another area of application for which the finding of shortest distances on polyhedral surfaces is of importance (Sharir and Schorr, 1984). Other areas of bio-physics and geo-physics would seem to provide possible areas of application of a generalized map makers algorithm.

Our interest is in obtaining a flat representation of the cortical surfaces of the brain, because the detailed maps of sensory and other neural data embedded in these surfaces are easiest to study and measure when they are shown on a plane surface.

This report describes the method we use to find an optimal quasi-isometry that maps an arbitrary curved surface into a plane. This mapping is optimal in the sense that it is derived from a variational principle that optimizes the overall fit between distances in both the curved and planar surfaces. The mapping is a quasi-isometry because it optimizes the fit of distances, over multiple scales, rather than, say, local angles (in which case it would be quasi-isothermal, or conformal).

We use a quasi-Newton-Raphson minimization, applying it to the distance matrix of inter-point distances up to some order of distance from each node. Minimal geodesic distances are calculated in the surface, and these form the initial distance matrix. The planar-distance matrix is initially generated from random numbers. Planar points are moved by the gradient of a "stress," which measures the quality of the isometry.

We have found that different random starting configurations converge to virtually identical final states. This finding indicates lack of "trapping" in local minima, and also indicates the robustness of our method. We also show that in general, local (i.e., nearest-neighbor-only) distances are insufficient for this problem. We base this conclusion on general arguments and on our own experience with local-distance problems.

Description of the algorithm

Given an arbitrary curved surface, we wish to "flatten" it. Because surfaces generally have non-zero Gaussian curvature, there is usually no isometry between the sur-

face and the corresponding plane (do Carmo, 1975). In other work (Kaplow and Schwartz, 1986), we describe ways to measure the mean and Gaussian curvature of a polyhedral surface.

We state the map-maker's problem in the following way: Given a polyhedral surface, determine the entire set of interpoint distances. This lower-triangular distance matrix is of size N(N-1)/2 for N nodes. We then determine a set of N points in the plane such that the corresponding distance matrix in the plane provides a best fit, in the least-squares sense, with the original distance matrix of the surface. This procedure should yield an optimal quasi-isometric mapping of the surface into the plane.

Distances in polyhedral surfaces

One difficulty with this algorithm is that until recently, no algorithms were known that can find distances on arbitrary (i.e., non-convex) polyhedral surfaces. Sharir and Schorr (1984) describe an algorithm that finds minimal distances in a convex polyhedral space, and other researchers (Mount, 1984, Mitchell and Papadimitrious, 1984, O'Rourke et al, 1984) have described algorithms to find distances on non-convex polyhedrons. These algorithms are complex, and we know of no actual implementations of them. Elsewhere we describe an algorithm we have developed that finds minimal distances in arbitrary polyhedral surfaces. In the present paper, we assume that we have obtained the distance matrix of the original surface by applying that algorithm to compute the discrete minimal distances (Wolfson and Schwartz, 1987).

Variational algorithm

Our mapping problem is very similar to the conventional "multi-dimensional" scaling problem, which is used to optimally map point sets from N to M dimensions, in which M is usually 1, 2 or 3, and in which N is usually large (e.g., 10-50). This is a common cluster-analysis procedure that is often used to visualize multivariate data sets (Sammon, 1969, Schiffman et al., 1981).

In the present case, because we are mapping between two-dimensional surfaces, we would expect good performance, especially because our surfaces, although highly convoluted or folded, do not usually represent "crumpled" spheres, but rather crumpled *sections* of spheres. In other words, our surfaces are not closed; and, as indicated by separate measurements, their integral mean curvature is not very large (Kaplow and Schwartz, 1986).

Description of the algorithm

Let the (minimal geodesic) distance in the surface be organized as a lower-triangular matrix d_{ij}. Let the (unknown) distances in the plane be organized as a lower-triangular matrix $\bar{d}_{ij}$. We pose the map-maker's problem as minimizing the least-squares goodness of fit between these two distance matrices. Thus, we propose to minimize the quantity L below:

$$L = \frac{1}{c} \sum_{i<j}^{i=N} \frac{\left[d_{ij} - \bar{d}_{ij}\right]^2}{\bar{d}_{ij}} \tag{1.0}$$

$$c = \sum_{i<j}^{i=N} \bar{d}_{ij} \tag{1.1}$$

Because the distance matrix contains all possible interpoint distances in this discrete problem, an optimal preservation of the distance matrix is equivalent to an optimal preservation of the metric structure of the original surface, in a planar representation.

We use the gradient-descent variational technique. Starting with a random planar point configuration, we calculate the initial distance matrix from these random points. Then we analytically calculate the gradient of equation (1.0) above, and move in the direction of the descending gradient by an amount scaled by the Hessian of equation

(1.0). This is the quasi-Newton Raphson technique. The quantity L in equation (1.0), which represents the goodness of fit of the metric structure of the data and planar-point sets, we term the "stress." Our approach minimizes this stress. Plots of stress as a function of the number of interations give some indication of how well this algorithm performs. In addition, testing the results with several random starting configurations, as shown in figure 2, can be used if there is any doubt about the validity of the solution.

Experiments with neighbor and next-neighbor distances

We implemented the algorithm of equation 1.0 above before we had finished its companion geodesic-distance-determination algorithm (Wolfson and Schwartz, 1987). Thus, our early experiments involved only neighbor distances (i.e., the edge-lengths of the original polyhedron) and next-neighbor distances (which we obtained easily by applying the law of cosines across two neighboring triangles). The results of these experiments were disappointing. When the algorithm was initialized with a random starting configuration, there generally was no convergence to the expected final state. Instead, the configurations we obtained seemed to be "folded." In other words, large regions had the right structure but were in the wrong position in the final planar configuration.

This result was easy to understand. When only short-range distances are available, little penalty is imposed on globally incorrect patches whose internal structural details are correct. This is because the penalty is essentially imposed only on the boundaries of the regions. The boundaries of patches of "diameter" **m** have a weight of $O(\mathbf{m})$, whereas the locally correct interior structure has a weight of $O(\mathbf{m}^2)$.

The only condition under which we were able to obtain successive flattenings with short-range distances was by supplying the algorithm with an initial state that had been relatively well flattened beforehand (i.e., through human intervention). Obviously, such a procedure cannot be called a flattening algorithm. However, by adding

longer-range distance terms, we successfully flattened polyhedra from random starting configurations, as described below.

Heuristic simplication of the algorithm

The size of the distance matrix (and hence the complexity of this algorithm) scales as N^2, where N is the number of nodes. The computer memory and CPU time required for large N are therefore prohibitive. We restricted the size of the neighborhood of distances represented for each node. This restriction provided a heuristic in which a "patch" of surface around each node contributed to the problem. Thus, our distance matrix became relatively sparse. By storing this data in a forest of binary-search trees, rather than a matrix, we achieved reasonable performance on a Sun 2 microprocessor system (which is roughly comparable to a VAX 750). The extent of these patches is an empirical problem. In our experience, a patch whose size is roughly 10% of the surface area of the entire problem yields good results. Thus, for example, as shown in Figure 3d, with O(1000) nodes, a patch of "diameter" of about 10 gave good results.

Examples of surface flattening

Example 1: Hemisphere

We generated a hemispherical shell, calculated the distance matrix analytically (for an eight neighborhood), and "flattened" it with our algorithm. The result is shown in Figure 1.

Example 2: Visual cortex

The surface of the visual cortex of a macaque monkey was digitized, triangulated, and flattened as in Figure 2. Figure 2 shows the "opercular" surface of visual cortex. The operculum (Latin for "roof") is an easily accessible part of the visual cortex. In fact, as shown in Figure 3b, the opercular cortex can be flattened physically with relatively little distortion. We studied the error term from physical flattening of the

opercular surface of cortex by making India-ink dots on cortex which we then flattened and filmed with a movie camera. Figure 3b is an picture from this series. For the opercular surface, once the isotropic expansion has been corrected, the physical flattening shear error is typically 5% to 10%. Our digital flattening errors(for the operculum), on the other hand, are typically in the 1% range.

The usefulness of digital flattening is clearer when we consider the entire striate cortex instead of just the operculum. Figure 3c is a computer-generated reconstruction of the entire cortex, viewed from below. Although the cortex is quite convoluted, this view shows that its surfaces are generally rather flat. In fact, it is easy to fold a sheet of paper into the petal-like shape shown in Figure 3c without tearing or stretching the paper.

Figure 3d shows the flattening of the entire cortex. The model in this example was a polyhedron consisting of about 2500 triangles. We flattened the model by using a 12-neighbor-deep distance around each node. Typical errors (defining error as the average local difference of edges in 3 dimensions and 2 dimensions) was typically in the range of a few percent.

Other applications

As noted earlier, this algorithm is a solution to the general map-maker's problem. The visual cortex is the largest and arguably the most convoluted of the cortical regions, and our cortical data seems to be rather more complex than the data in other common instances of the problem. The good results we have obtained with this data encourage us to think that our methods can deal competently with a variety of problems related to the mapping of surfaces. In particular, we believe that problems in robot-motion planning and autonomous vehicle-navigation might yield to an application of this algorithm.

Figure Captions

Figure 1. A hemispherical shell was flattened by our algorithm. This figure shows the 3D wire-frame model of the original surface and also the flattened version of the surface. A "neighborhood" extending at least eight nodes from each node determined the "patch" of surface for which we computed the distances.

Figure 2.

A section of the "operculum" of visual cortex, reconstructed from serial sections of monkey brain, was flattened by our algorithm. The three different versions of the planar surface were obtained in successive runs, with different random starting configurations. Because the algorithm does not constrain the angle of rotation, the solution is determined only up to a rotation and reflection. However, size is preserved. The three solutions are virtually identical, suggesting that no local minima that might be present have obscured the solution. This suggestion is reinforced by the plot of the stress as a function of iterations, shown at the lower right. In this parameter (which determines the overall least-squares goodness of fit) there is some large oscillation early in the run; but after about 100 iterations, all three runs converge to the same stress and to the same geometric solution. The curved appearance of these figures is an illusion caused by the curved nature of the original polyhedron. In fact, all three of these examples are of course planar representations.

For this model, minimal distances were calculated usuing the algorithm of Wolfson and Schwartz (1986). Use of an eight-node neighborhood of the 600-node model limited the size of the distance matrixes.

Figure 3a.

NOTE: ORIGINAL FIGURE IS GRAY SCALE FOR JOURNAL REPRODUCTION.

A block of monkey brain. The smooth roof (operculum) of visual cortex is the bullet-shaped region at the left.

Figure 3b.

NOTE: ORIGINAL FIGURE IS GRAY SCALE FOR JOURNAL REPRODUCTION.

The opercular visual cortex is physically flattened by being pressed between glass plates while being filmed with an 8mm movie camera. The India-ink dots visible on the cortex were used to measure the distortion caused by physical flattening.

Figure 3c.

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A computer-generated reconstruction of the entire striate cortex, viewed from the bottom. The opercular surface is away from the viewer, while the convoluted "calcarine" cortex appears as a flower-petal shape closer to the viewer.

Figure 3d. The entire striate cortex from Figure 3c, as flattened by our algorithm. Greater densities of triangles in the model indicate the regions of higher curvature on the cortical surface.

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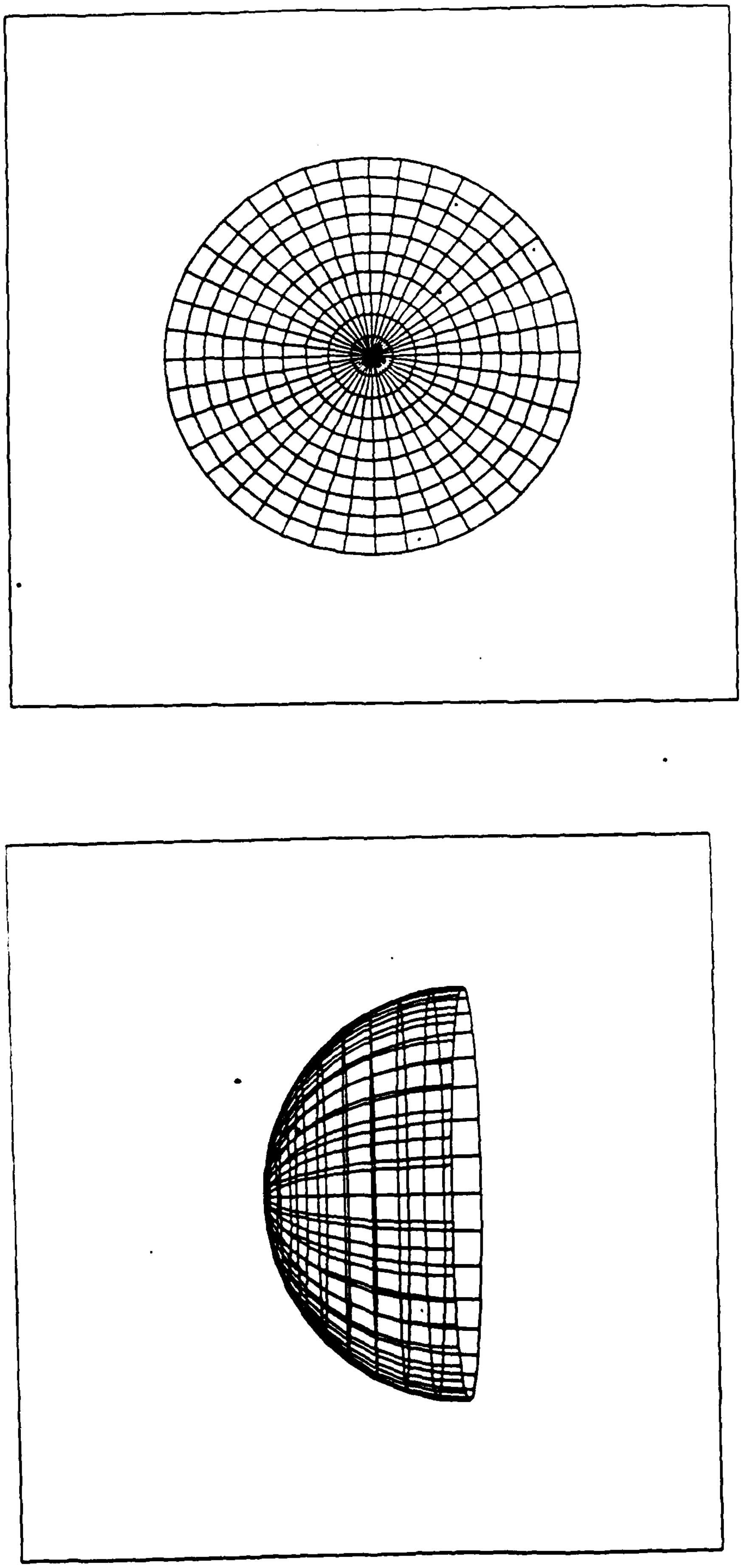
FIGURE 1



FIGURE

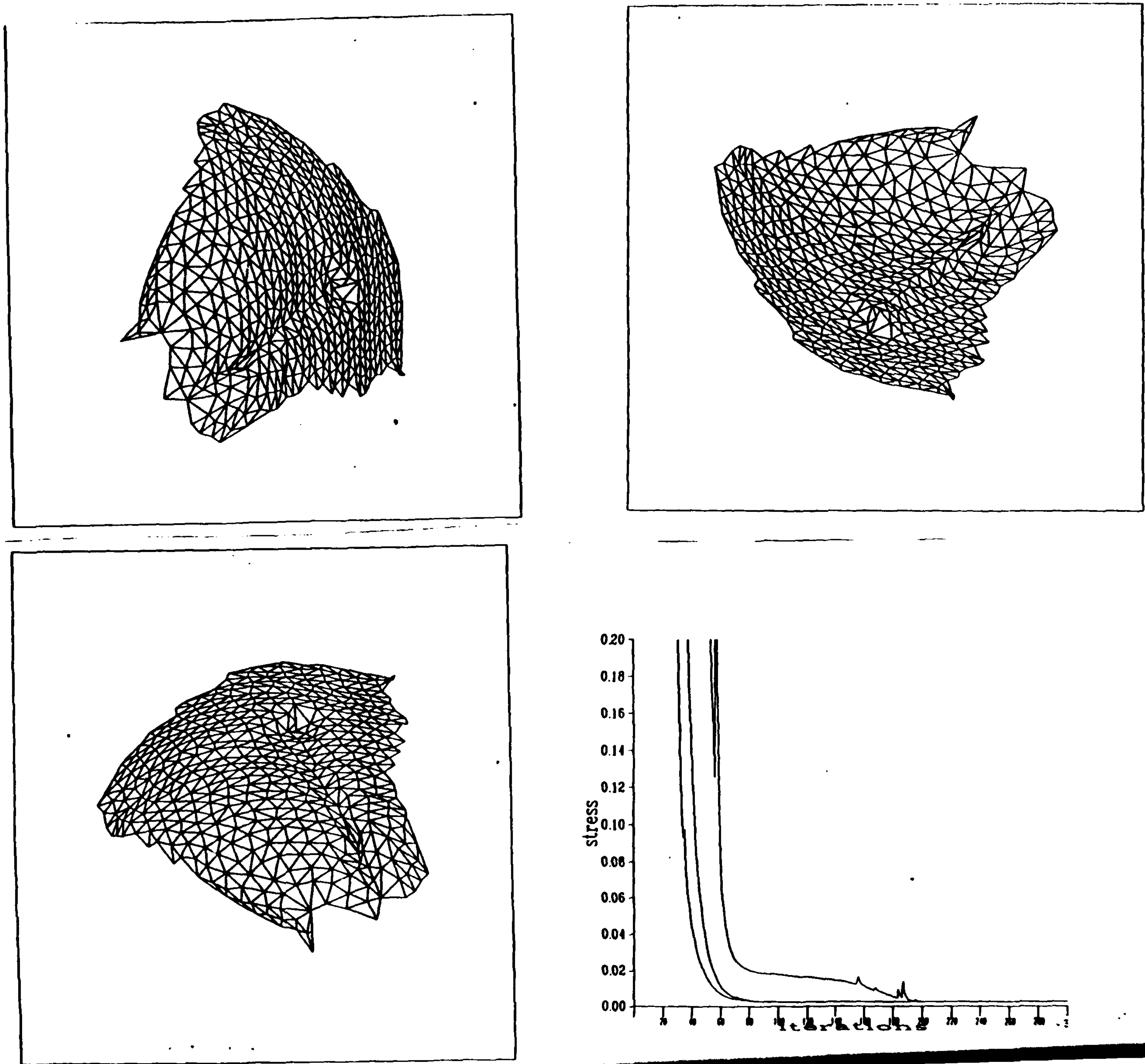


FIGURE 3A

FIGURE 3B

FIGURE 3D

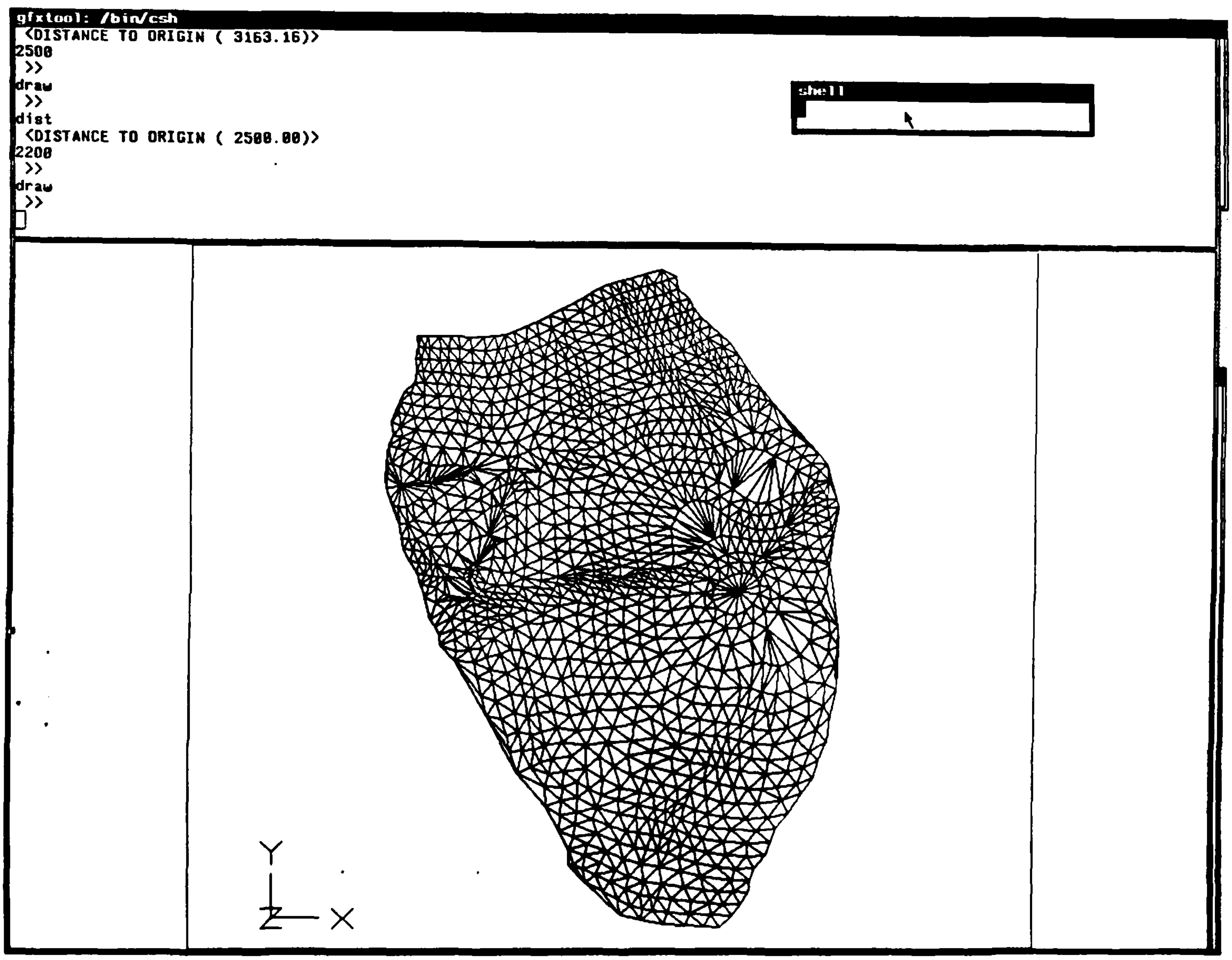

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FIGURE 3C